Nusselt Values for Estimating Turbulent Liquid Metal Heat Transfer in Noncircular Ducts

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A simple expression for estimating the turbulent forced-convection heat transfer performance of liquid metals flowing through noncircular ducts is presented. This equation requires the knowledge of the slug Nusselt number evaluated for the specific geometry and for the pertinent boundary conditions. Such Nusselt values are presented herein for a number of technically important geometries. One check on the heat transfer prediction given by Equation (4) is in the case of an annular duct with constant heat flow through the outer wall with the inner wall insulated, for which experimental data exist. The prediction agrees within 20% with the experimental data.

Several possible boundary conditions that may exist in noncircular cross sections are thoroughly discussed, and it is hoped that as a result this paper may serve to clarify some of the confusion existing in the literature.

Liquid metals continue to be of considerable interest as primary coolants in nuclear power plants. Examples may be found in the sodium reactor experiment of Atomics International and in the fast breeder reactor being planned by Power Reactor Development Company, both of which use sodium as the coolant. In such applications the geometry of the cooling passages through which the liquid metal flows is determined by engineering and nuclear considerations which frequently dictate noncircular shapes. The purpose of the present paper is to provide a method of estimating the convective heat transfer and wall-temperature distribution for turbulent flow of liquid metals through passages having such noncircular cross sections.

NUSSELT NUMBER EQUATION FOR TURBULENT LIQUID METAL FLOW

It has been pointed out by Lyon (*1*), Claiborne (*2*), and others that turbulent forced-convection heat transfer performance of liquid metals may be estimated by an appropriate combination of the separate contributions due to molecular conduction and turbulent exchange. Since the velocity profile in turbulent flow approximates a slug flow profile (constant velocity across the passage cross section), it is reasoned that the convective contribution due to molecular conduction should be obtainable from the energy-equation solution by utilizing the actual temperature or heat-flow boundary conditions but assuming the slug velocity distribution. The resulting slug Nusselt number is then decreased somewhat to account for the fact that the actual velocity profile is not uniform but falls off in the region of the walls. The turbulent contribution must then be added to the modified slug Nusselt number to give an expression of the following form for the actual Nusselt number:

$$Nu = C(Nu_s) + \text{turbulent contribution} \qquad (1)$$

where Nu = Estimated Nusselt number for turbulent liquid metal flow

Nu_s = slug Nusselt number

C = correction factor for actual velocity profile

Specifically, if the circular tube is considered, the Lyon equation (*1*) which is based on the results of extensive calculations using the momentum analogy approach is expressed as

$$Nu = 7.0 + 0.025\,(Pe)^{0.8} \qquad (2)$$

Since the Nusselt number for slug flow under the same boundary conditions is 8.0, comparison of the Lyon equation with Equation (1) yields a value of C of $7/8$ and a turbulent contribution of $0.025\,(Pe)^{0.8}$.

On this basis Seban and Shimazaki (*3*) proposed the following expression for the case of constant wall temperature for the circular tube where the slug Nusselt number is 5.8:

$$Nu = \tfrac{7}{8}(5.8) + 0.025\,(Pe)^{0.8}$$

or

$$Nu = 5.0 + 0.025\,(Pe)^{0.8} \qquad (3)$$

This expression agrees with the results obtained by the momentum analogy (*3*).

A recent survey by Lubarsky and Kaufman (*4*) of all available liquid metal pipe-flow data indicates that Equations (2) and (3) yield results which are somewhat higher than experimental data; however, by use of the survey detailed in reference *4*, the constants appearing in Equations (2) and (3) were revised downward to agree with the pipe-flow data. This was done and the following equation is recommended by the present authors for an estimate of the convective heat transfer for a liquid metal (Prandtl number less than 0.1) flowing turbulently through a duct circular or noncircular in cross section:

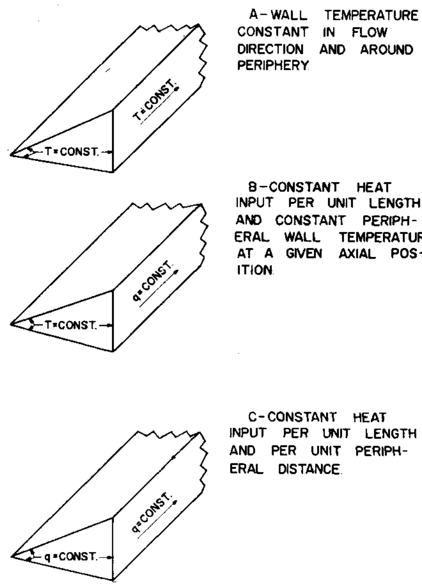

Fig. 1. Boundary conditions of importance for noncircular ducts.

$$Nu = \tfrac{2}{3}Nu_s + 0.015\,(Pe)^{0.8} \qquad (4)^*$$

The main purpose of this paper is to present all available slug Nusselt values for ducts with noncircular cross sections. In the interest of completeness, previously reported values are included, and, in addition, a number of new solutions are given. It should be emphasized that Equation (4) is not valid for ordinary fluids such as air or water but is restricted to liquid metals.

In those cases where the wall boundary condition is one of constant heat flux everywhere, the knowledge of the resulting wall-temperature distribution is of more importance than the Nusselt numbers. For several geometries of importance such wall-temperature distributions are given for the limiting case of low turbulent Reynolds numbers. All results are presented in a form convenient for engineering calculations.

BOUNDARY CONDITIONS IN NONCIRCULAR DUCTS

Considerable confusion is apparent in the literature concerning the possible boundary conditions and their implica-

*In the original manuscript the constant 0.019 appeared rather than 0.015. The revision downward was suggested by H. C. Claiborne and brings the equation into better agreement with the experimental data.

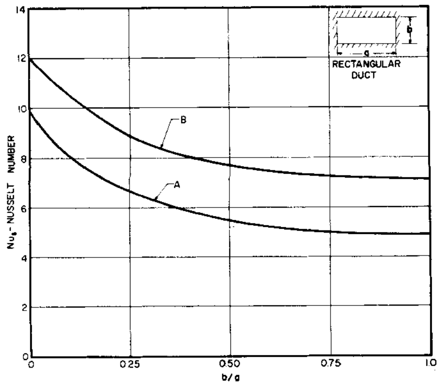

Fig. 2. Slug Nusselt numbers for rectangular duct.

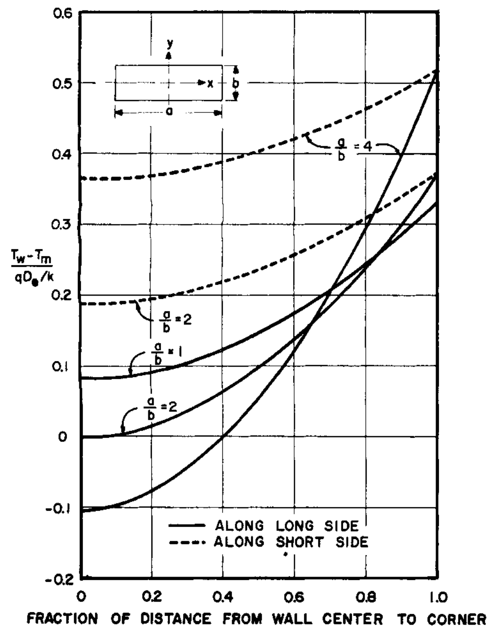

Fig. 3. Limiting wall-temperature distribution for rectangular duct with constant heat flux everywhere.

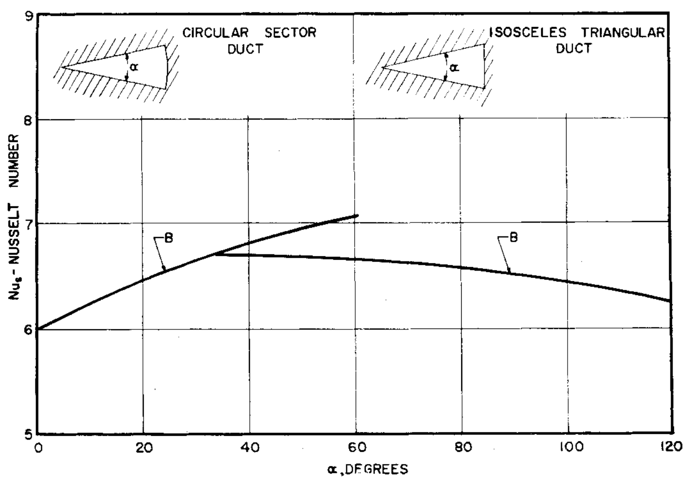

Fig. 4. Slug Nusselt numbers for circular-sector duct and isosceles-triangular duct.

tions in noncircular ducts. This has been previously pointed out by Eckert, Irvine, and Yen (*5*). For a circular pipe no ambiguity is encountered, and two boundary conditions of major importance are usually considered: constant wall temperature and constant heat input per unit length. If attention is restricted to conditions of established hydrodynamic and thermal fields, the constant-wall-temperature case requires the fluid temperature gradually to approach the wall value in the flow direction. In the second case of constant heat input each increment of surface area transfers the same heat flow to the fluid and consequently the wall and fluid temperatures increase linearly with length at the same rate. Owing to the symmetry of the circular tube, the wall temperature is constant around the circumference at any cross section and the thermal properties of the wall material do not enter into any circumferential considerations.

On the other hand, if a geometry is considered which lacks circular symmetry, such as the triangle, rectangle, or ellipse, the situation becomes more complex. In these cases with constant heat generation a peripheral temperature gradient can exist at a given cross section and the thermal properties of the wall can enter into the problem. In addition, the source of the heat generation must be specified, i.e., on the outer or inner surface of the wall or within the wall itself. Three tractable boundary conditions of technical interest may therefore be distinguished for ducts with noncircular cross sections, as shown on Figure 1:

1. The wall temperature remains constant everywhere (designated throughout the paper as case *A*).
2. The heat input per unit length is constant and the wall temperature is constant around the periphery at a given axial position. This condition requires the local heat input to the fluid to vary around the periphery. In practice this will occur if the heat is uniformly generated at the outer duct wall and if the wall conduction is very large in the peripheral direction. In contradistinction to case 3 below, this may be called the thick-walled duct with uniform heat generation (case *B*).
3. The heat input per unit area is constant in both the peripheral and flow direction. This condition requires the wall temperature to vary around the periphery at a given axial position and can be realized in practice in a very thin-walled duct (zero peripheral wall conductivity) where the heat is uniformly generated at the outer duct wall (case *C*).

It may be noted that cases *B* and *C* are identical for circular tubes and infinite slots.

In addition to these cases, if the geometry of interest has independent boundary surfaces such as an annulus, other boundary conditions may be considered:

1. Constant heat input on one wall with the other wall insulated.
2. Constant heat input on both walls. This condition requires the temperature of the inner and outer walls to be different at the same cross section.
3. Constant heat input per unit length with equal peripheral wall temperatures at every cross section. This condition requires a different heat rate from the inner and outer surfaces to the fluid.
4. Constant wall temperature everywhere.
5. Constant wall temperature on one wall with the other wall insulated.

Although other combinations are possible, these will serve to demonstrate the many different boundary conditions which may be encountered. In particular it should be pointed out that care must be taken to utilize the correct slug-flow solution (i.e., the one corresponding to the correct boundary conditions) in evaluating Equation (4).

DISCUSSION OF THE SLUG-FLOW DIFFERENTIAL EQUATION

For a fluid in steady duct flow with constant property values under the conditions of fully developed velocity and temperature fields, the energy equation assumes the following form:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \frac{w}{\kappa}\frac{\partial T}{\partial z} \qquad (5)$$

The velocities in the x and y directions perpendicular to the flow are zero.

The axial conduction term $\partial^2 T/\partial z^2$ has been omitted in Equation (5) as its contribution is negligible in case *A*. For cases *B* and *C* the axial conduction term is identically zero under the fully developed thermal conditions assumed in the analysis.

In the following solutions the velocity w is taken as constant at any cross section or longitudinal position. The previously defined cases *A*, *B*, and *C* will be discussed in more detail.

Case A: Wall Temperature Everywhere Constant

Under these conditions the fluid longitudinal temperature gradient $\partial T/\partial z$

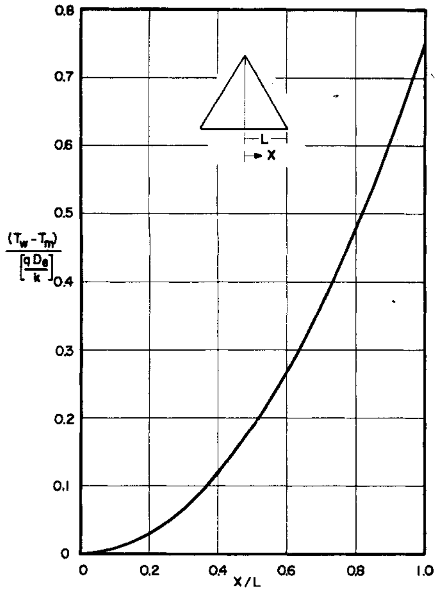

Fig. 5. Limiting wall-temperature distribution for equilateral-triangular duct with constant heat flux everywhere.

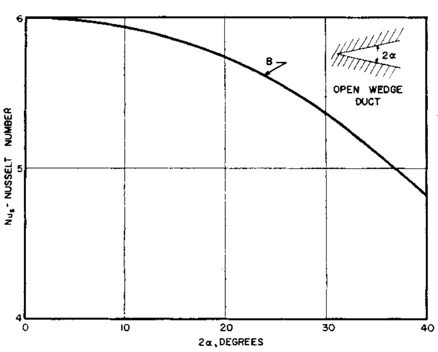

Fig. 6. Slug Nusselt numbers for infinite open-wedge duct for boundary condition *B*.

changes in the downstream direction as the mean fluid temperature approaches the wall value. One of the earliest reported solutions for this case was by Graetz (*6*) for slug flow through a circular pipe. Fortunately, the differential equation (5) also describes the temperature field as it changes with time in a solid of the same shape and as a result some useful slug-flow-convection solutions have appeared in the heat-conduction literature for the constant-wall-temperature boundary condition. For example, Carslaw and Jaeger (*7*) present the temperature distributions for the rectangle and annulus, and Jaeger (*8*) treats the circular sector and open wedge. A number of these solutions are translated into a form usable for engineering calculations and are presented in a later section.

Case B: Constant Heat Input Per Unit Length, Constant Peripheral Wall Temperature

Under these conditions a simple heat balance will show the longitudinal temperature gradient to be constant. The differential equation (5) reduces to Poisson's equation in two dimensions; i.e.,

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \text{constant} \qquad (6)$$

This equation has been extensively studied in mathematical physics as it describes a great number of physical phenomena. In particular, Equation (6) describes the velocity field in the laminar flow of a fluid under the conditions of a fully developed velocity field. Eckert and Irvine (*9*) recently collected available velocity-distribution solutions for noncircular cross sections and in addition have obtained solutions for new shapes. Their results are essentially reported as the product of the friction factor f and the Reynolds number where the friction factor is defined as

$$f = \frac{\frac{\partial P}{\partial z}}{\rho \bar{w}^2 / 2D_e} \qquad (7)$$

Inspection of the governing differential equations, the friction-factor definition,

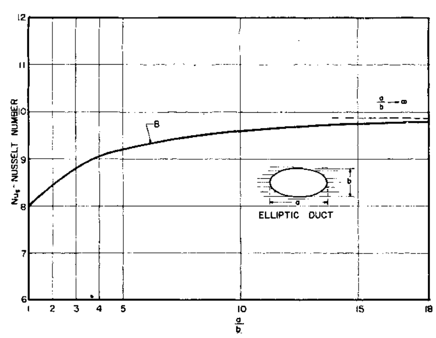

Fig. 7. Slug Nusselt numbers for elliptic duct for boundary condition *B*.

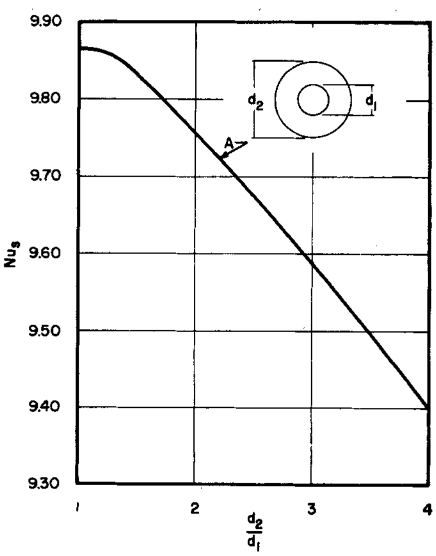

Fig. 8. Slug Nusselt numbers for annular duct with constant wall temperature, boundary condition *A*.

and the definition of the slug Nusselt number yields the following interesting relation between the friction factor and the slug Nusselt number:

$$Nu_s = \frac{f \cdot Re}{8} \qquad (8)$$

The obvious result is that *all calculations* by Eckert and Irvine (*9*) may be directly converted to slug Nusselt numbers.

Case C: Constant Heat Input Per Unit Surface Area Everywhere on Boundary Walls

This boundary condition requires the temperature gradient normal to the inner wall surface to be constant at all wall locations. Because of constant longitudinal heat addition, the differential equation is once again in the form of Poisson's equation, Equation (6). While case C is considerably easier to approach in practice than case B (case C being a thin-walled duct with constant heat addition at the exterior of the duct), the solution of the differential equation is more difficult.

In both previous cases the slug Nusselt number was the heat transfer parameter of primary interest. In this case where the heat flux is already specified, the peripheral-wall-temperature distribution is generally of greater concern, as structural considerations normally dictate a maximum possible operating temperature. Limiting wall temperature distributions therefore will be given for case C; it should be emphasized, however, that such results are strictly valid only for the slug-flow velocity distribution with heat transfer only by molecular conduction. Thus these limited temperature distributions approximate conditions to be expected for turbulent flow at Reynolds numbers slightly above the critical value. As the Reynolds number is increased, turbulent exchange will have a mitigating effect on the wall temperature gradients. However, the solutions point out the locations of maximum temperature.

PRESENTATION OF SLUG NUSSELT NUMBERS AND WALL-TEMPERATURE DISTRIBUTIONS

The values of slug Nusselt numbers and wall-temperature distributions will be given in this section for all three boundary conditions under the heading of the particular geometry.

Rectangular Duct

For this geometry the solutions for boundary conditions A and B are shown in Figure 2. The constant-wall-temperature boundary condition, case A, is treated by Carslaw and Jaeger (*7*) and case B is treated by Timoshenko (*10*) and by Claiborne (*2*) although some confusion with respect to boundary conditions A and B is evident in this last reference.

The wall-temperature distribution for case C from this reference (*2*) is given below and plotted in Figure 3.

$$\left[\frac{T_w - T_m}{qD_e/\kappa}\right]_{y=b/2} = \frac{x^2}{aD_e} + \frac{b}{4D_e} - \frac{a+b}{12D_e} \quad (9)$$

$$\left[\frac{T_w - T_m}{qD_e/\kappa}\right]_{x=a/2} = \frac{a}{4D_e} + \frac{y^2}{bD_e} - \frac{a+b}{12D_e} \quad (10)$$

The center of the coordinate axis is taken as the geometric center of the rectangle and x and y are measured from this point.

This dimensionless presentation for case C was chosen since the hydraulic diameter D_e, the fluid thermal conductivity k, and the heat flux q will necessarily be specified, and the local fluid bulk temperature T_m is readily calculated (since q is known and the fluid temperature at the duct entrance will be known), and therefore the only unknown is T_w. It is apparent from Figure 3 that the maximum wall temperature will occur in the corners.

It is of interest to note that the rectangle includes as special cases (1) the square duct when the two sides are of equal magnitude and (2) the infinite slot when the ratio of the sides goes to zero.

Isosceles-triangular Duct and Circular-sector Duct

The triangular-shaped duct is frequently encountered in practice, and so these solutions are particularly useful. The circular-sector duct is included in this section as it should closely approximate the isosceles triangle in those cases where the apex angle α is small. The constant-wall-temperature case (case A) has been treated (*8*) for the circular-sector duct, but no comparable solution was found for the isosceles-triangular duct. For boundary condition B the results are shown in Figure 4, which presents both the circular-sector and the isosceles-triangle values. The isosceles-triangular-duct values were obtained by converting an approximate velocity-distribution solution previously given by Nuttall (*11*), and these results become somewhat questionable at small angles. It is therefore recommended that the approximate isosceles-triangular-duct results be used only above 30 deg. and the circular sector utilized at smaller angles. It is of interest that the slug Nusselt value exhibits only small change over a wide range of apex angles.

The constant heat-flux boundary condition, case C, has been discussed for the circular-sector duct (*2*) but considerable work would be required to convert the results into an engineering form. No comparable solution for the general isosceles-triangular duct is known, although the special case of the equilateral triangle is tractable (*2*). The resulting wall-temperature distribution for the equilateral triangle is shown in Figure 5. As expected, the maximum wall temperature occurs in the corners. It is of additional interest that at the midpoint of the walls the wall temperature is exactly equal to the bulk fluid temperature.

It has been pointed out in (*12*) that in ducts with triangular cross sections, it is possible to have laminar flow in the corners when it might normally be expected that turbulent flow exists. Some caution must therefore be exercised in applying turbulent relations to ducts having narrow corner regions.

Open Wedge or Infinite Wedge

This particular geometry is of interest even though it will seldom be found in practice, as it has been shown in reference *12* that for small angles the effect of the base wall on the fluid near the apex becomes negligible. It is therefore possible to investigate the heat transfer conditions in the critical apex regions with expressions which are considerably simpler than those for the circular sector.

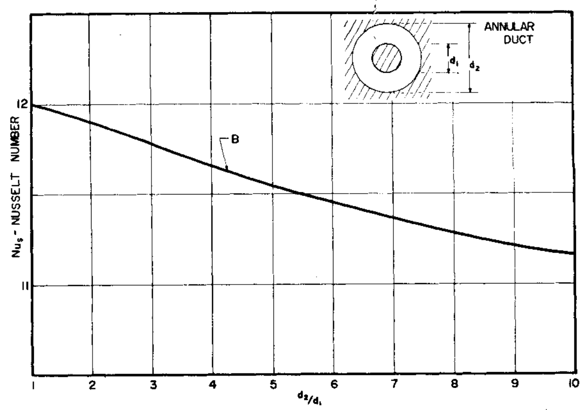

Fig. 9. Slug Nusselt numbers for annular duct for boundary condition *B*.

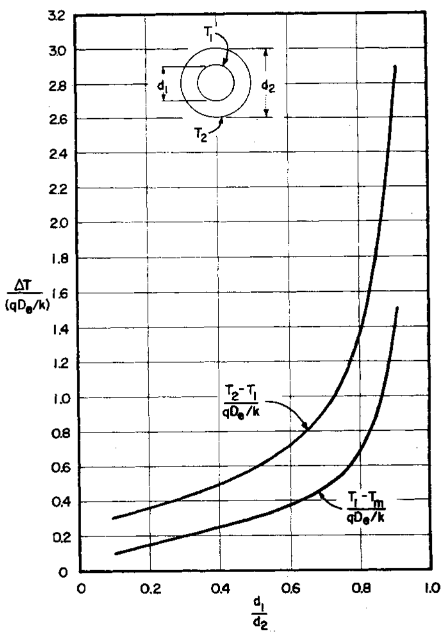

Fig. 10. Limiting wall-temperature distribution for annular duct with constant heat flux everywhere, boundary condition *C*.

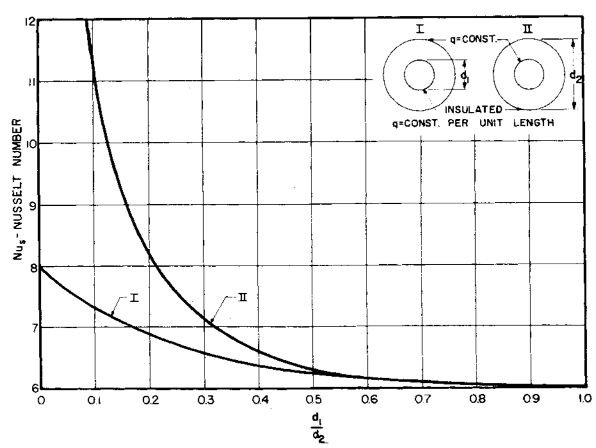

Fig. 11. Slug Nusselt numbers for annular duct with constant heat input on one wall with the other wall insulated.

TABLE 1. SUMMARY OF SLUG-FLOW SOLUTIONS FOR VARIOUS GEOMETRIES

Geometry	Boundary condition (See Fig. 1)	Slug Nusselt numbers	Slug-flow Wall-temperature distribution
Rectangle	*A*	Fig. 2	
	B	Fig. 2	
	C		Eq. (10), Fig. 3
Circular Sector	*A*	Available in Ref. (8)	
	B	Fig. 4	
	C		Available in ref. (2)
Isosceles triangle	*A*	No Solution Obtained	
	B	Fig. 4	
	C		No general solution obtained
Equilateral triangle	*A*	No Solution Obtained	
	B	Table II and Fig. 4	
	C		Fig. 5
Open wedge	*A*	Available in Ref. (8)	
	B	Fig. 6	
	C		No solution obtained
Ellipse	*A*	No Solution Obtained	
	B	Fig. 7	
	C		Available in ref. (2)
Annulus	*A*	Fig. 8	
	B	Fig. 9	
	C		Fig. 10
	Constant heat input on one wall with the other wall insulated	Fig. 11	

The solution to case *A* is given in reference *9*, while the slug Nusselt numbers for case *B* are given in Figure 6. No solution was obtained for case *C*.

Ellipse

For the ellipse the slug Nusselt number for case *B* is shown in Figure 7. No solution for cases *A* and *C* has been found although reference *2* has an expression for the solution to case *C* in the form of unevaluated Fourier coefficients.

Annular Duct

As discussed in an earlier section, a large number of boundary conditions are possible for this geometry. Figure 8 presents the slug Nusselt values for the case where both walls are at constant temperature. The Nusselt value for the case of constant heat input per unit length with both inner and outer walls at the same temperature at a given axial position is presented in Figure 9; for this case the heat input per unit area is not the same at both wall surfaces. Figure 10 presents the wall-temperature variation for the case where there is uniform heat input per unit area from every element of both wall surfaces. Finally, the case of constant heat input on one wall with the other wall insulated is shown in Figure 11.

SUMMARY AND RESULTS

The summary of the available slug-flow solutions as previously discussed is presented in Table 1, which provides a ready reference as to the availability and location of essentially all technically important noncircular geometries. For additional convenience a second table, Table 2, summarizes slug Nusselt values for simple geometries of technical importance. All values appearing in Table 2 are special cases of the more general geometries covered in Table 1.

TABLE 2. SLUG NUSSELT NUMBERS FOR SIMPLE GEOMETRIES

Geometry	Boundary condition (See Fig. 1)	Slug Nusselt number
Circle	*A*	5.80
	B	8.0
Square	*A*	$\pi^2/2 = 4.93$
	B	7.03
Equilateral triangle	*A*	
	B	6.67
Infinite slot	*A*	$\pi^2 = 9.87$
	B	12
Infinite slot with one wall insulated	*A*	$\pi^2/2 = 4.93$
	B	6
90-deg. isosceles triangle	*A*	
	B	6.55

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The authors acknowledge their debt to James Yen and William Welsh for carrying out many of the lengthy calculations reported herein.

NOTATION

English Letter Symbols

a = length of long side of rectangle
b = length of short side of rectangle
C = constant
c_p = specific heat at constant pressure
D_e = equivalent or hydraulic diameter $\left(\frac{4 \text{ cross section area}}{\text{perimeter}}\right)$
d = diameter of annulus
f = friction factor, defined in Eq. (7)
h = heat transfer coefficient, defined as $q/T_w - T_m$
k = thermal conductivity
p = pressure
q = heat rate per unit area
T = temperature
w = velocity in z direction
$\bar{w}$ = mean velocity for actual laminar flow profile as utilized in Eq. (7)
x, y, z = coordinate directions

Subscripts

s = slug
m = mean or bulk conditions
e = hydraulic or equivalent
w = wall conditions

Dimensionless Ratios

Nu = Nusselt number, hD_e/k
Pr = Prandtl number, $\mu c_p/k$
Pe = Peclet number, $\rho c_p \bar{w} D_e/\kappa$
Re = Reynolds number $\rho \bar{w} D_e/\mu$

Greek Symbols

α = angle
κ = thermal diffusivity
ρ = density
μ = dynamic viscosity

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